Całki oznaczone.

1. Znaleźć całki (w [.] są wyniki): a) $\int\limits_{-4}^{-2}\frac{dx}{x^2+6x+10}$ $\left[\frac{\pi}{2}\right]$ b) $\int\limits_0^1 x\ln(x^2+1)\,dx$ $\left[\ln 2-\frac{1}{2}\right]$ (p.części)

c) $\int\limits_0^{\frac{\pi}{4}}\mathrm{tg}\,x\,dx$ $\left[\ln\sqrt{2}\right]$ d) $\int\limits_0^{\frac{\pi}{2}}\sin^3 x\,dx$ $\left[\frac{2}{3}\right]$ e) $\int\limits_0^{\frac{\pi}{2}}\sin^4 x\cdot\cos^3 x\,dx$ $\left[\frac{2}{35}\right]$ f) $\int\limits_0^{\frac{\pi}{3}}\mathrm{tg}^3 x\,dx$ $\left[\frac{2}{3}-\ln 2\right]$

g) $\int\limits_0^{\sqrt{\frac{\pi}{2}}} x\sin x^2 dx$ $\left[\frac{1}{2}\right]$ h) $\int\limits_0^{\frac{\pi}{3}}\cos^2 x\,dx$ $\left[\frac{\sqrt{3}}{8}+\frac{\pi}{6}\right]$ i) $\int\limits_0^{\sqrt{3}}\frac{x\,dx}{\sqrt{x^2+1}}$ [1] j) $\int\limits_0^5\frac{x+2}{\sqrt{x+4}}dx$ $\left[\frac{26}{3}\right]$

k) $\int\limits_0^{\frac{\pi}{2}}\frac{\cos x}{\sqrt{2+\sin x}}dx$ $\left[2\sqrt{3}-2\sqrt{2}\right]$ l) $\int\limits_{\sqrt{2}}^{\sqrt{5}}\frac{x}{\sqrt{x^2-1}}dx$ $[1]$ m) $\int\limits_{-\frac{1}{2}}^{1}\frac{dx}{\sqrt{8+2x-x^2}}$ $\left[\frac{\pi}{6}\right]$

l) $\int\limits_1^{\sqrt{3}} x\,\mathrm{arc\,tg}\frac{1}{x}dx$ $\left[\frac{\pi}{12}+\frac{1}{2}\cdot(\sqrt{3}-1)\right]$.

Rozwiązania niektórych zadań.

1. b) $$\int\limits_0^1 x\ln(x^2+1)\,dx=\left[\begin{matrix}u=\ln(x^2+1) & u'=\frac{2x}{x^2+1}\\ v'=x & v=\frac{x^2}{2}\end{matrix}\right]=\frac{x^2}{2}\ln(x^2+1)\Big|_0^1-\int\limits_0^1\frac{x^3}{x^2+1}dx=$$

$$=\frac{1}{2}\ln 2-\left[\int\limits_0^1 x\,dx-\int\limits_0^1\frac{x}{x^2+1}dx\right]=\frac{1}{2}\ln 2-\frac{x^2}{2}\Big|_0^1+\frac{1}{2}\ln(x^2+1)\Big|_0^1=\ln 2-0{,}5$$

f) $$\int\limits_0^{\frac{\pi}{3}}\mathrm{tg}^3 x\,dx=\left|\begin{matrix}t=\mathrm{tg}\,x\\ \mathrm{arctg}\,t=x\\ \frac{dt}{1+t^2}=dx\end{matrix}\right|=\int\limits_0^{\sqrt{3}}\frac{t^3}{1+t^2}dt=\int\limits_0^{\sqrt{3}}\frac{t^3+t-t}{1+t^2}dt=\int\limits_0^{\sqrt{3}}\frac{t^3+t}{1+t^2}dt-\tfrac{1}{2}\int\limits_0^{\sqrt{3}}\frac{2t}{t^2+1}=\int\limits_0^{\sqrt{3}}t\,dt-\tfrac{1}{2}\ln(t^2+1)\Big|_0^{\sqrt{3}}=$$

x	0	$\frac{\pi}{3}$
t	0	$\sqrt{3}$

$$=\frac{t^2}{2}\Big|_0^{\sqrt{3}}-\tfrac{1}{2}\ln 4-\tfrac{1}{2}\ln 1=\frac{3}{2}-\ln 2\,.$$

g) $$\int\limits_0^{\sqrt{\frac{\pi}{2}}}x\sin x^2 dx=\left[\begin{matrix}x^2=t\\ 2x dx=dt\end{matrix}\right]=\tfrac{1}{2}\int\limits_0^{\frac{\pi}{2}}\sin t\,dt=-\tfrac{1}{2}\cos t\Big|_0^{\frac{\pi}{2}}=0+\tfrac{1}{2}\cdot 1=\frac{1}{2}\,.$$

x	0	$\sqrt{\frac{\pi}{2}}$
t	0	$\frac{\pi}{2}$

h) $$\int\limits_0^{\frac{\pi}{3}}\cos^2 x\,dx=\left[\cos^2 x=\frac{1+\cos 2x}{2}\right]=\tfrac{1}{2}\int\limits_0^{\frac{\pi}{3}}(1+\cos 2x)dx=\tfrac{1}{2}x\Big|_0^{\frac{\pi}{3}}+\tfrac{1}{4}\sin 2x\Big|_0^{\frac{\pi}{3}}=\frac{\pi}{6}+\frac{1}{4}\sin\frac{2\pi}{3}=\frac{\pi}{6}+\frac{1}{4}\cdot\frac{\sqrt{3}}{2}=$$

$$=\frac{\pi}{6}+\frac{\sqrt{3}}{8}\,.$$

i) $\int_0^{\sqrt{3}} \frac{xdx}{\sqrt{x^2+1}} = \begin{bmatrix} \sqrt{x^2+1} = t \\ x^2+1 = t^2 \\ xdx = tdt \end{bmatrix} = \int_1^2 \frac{t\,dt}{t} = t\,\Big|_1^2 = 2-1 = 1$

x	0	$\sqrt{3}$
t	1	2

k) $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{2+\sin x}}dx = \begin{bmatrix} t = \sqrt{2+\sin x} \\ t^2 = 2+\sin x \\ 2tdt = \cos x dx \end{bmatrix} = \int_{\sqrt{2}}^{\sqrt{3}} \frac{2tdt}{t} = 2t\Big|_{\sqrt{2}}^{\sqrt{3}} = 2\sqrt{3} - 2\sqrt{2}$.

x	0	$\frac{\pi}{2}$
t	$\sqrt{2}$	$\sqrt{3}$

m) $\int_{-\frac{1}{2}}^{1} \frac{dx}{\sqrt{8+2x-x^2}} = \int_{-\frac{1}{2}}^{1} \frac{dx}{\sqrt{9-(x-1)^2}} = \frac{1}{3}\int_{-\frac{1}{2}}^{1} \frac{dx}{\sqrt{1-(\frac{x-1}{3})^2}} = \arcsin(\frac{x-1}{3})\Big|_{-\frac{1}{2}}^{1} = \arcsin 0 - \arcsin(-\frac{1}{2}) = 0 - \left(-\frac{\pi}{6}\right) = \frac{\pi}{6}$.